

7 Graphing Advanced Rational Equations

Warmup

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1. $(x^3 - 1) \div (x - 1)$

$$x^2 + x + 1$$

2. $(y^3 + 27) \div (y + 3)$

$$y^2 - 3y + 9$$

3. $(64x^3 + 27) \div (4x + 3)$

$$16x^2 - 12x + 9$$

4. $(27x^3 + 8) \div (3x + 2)$

$$9x^2 - 6x + 4$$

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Domain

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$$f(x) = \frac{1}{x+2}$$

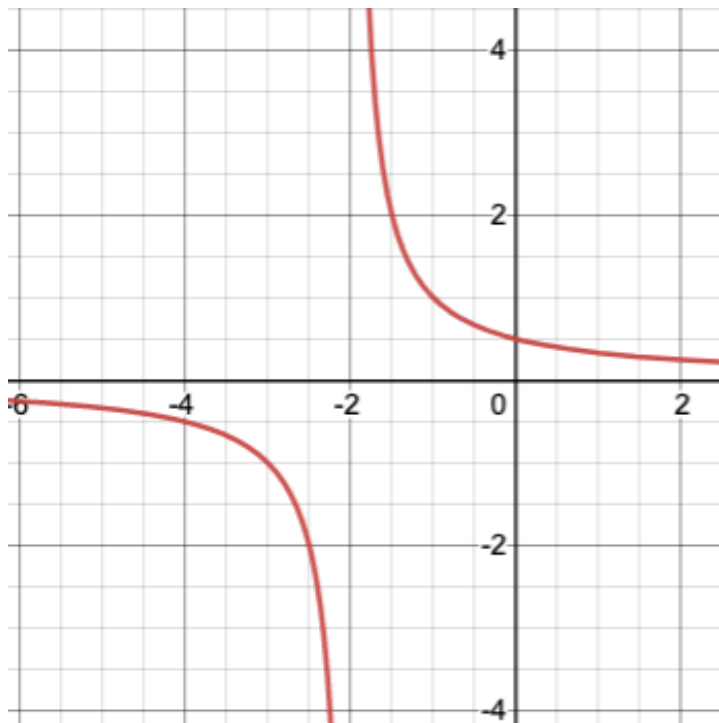
Vertical or horizontal asymptotes?

Vertical ($x = -2$), Horizontal ($y = 0$)

What's the behavior of the graph as it approaches the vertical asymptote from the right/left?

$$\lim_{x \rightarrow -2^-} f(x) = -\infty \quad \text{From left}$$

$$\lim_{x \rightarrow -2^+} f(x) = +\infty \quad \text{From right}$$



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Domain

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$$f(x) = \frac{1}{(x+2)(x-3)}$$

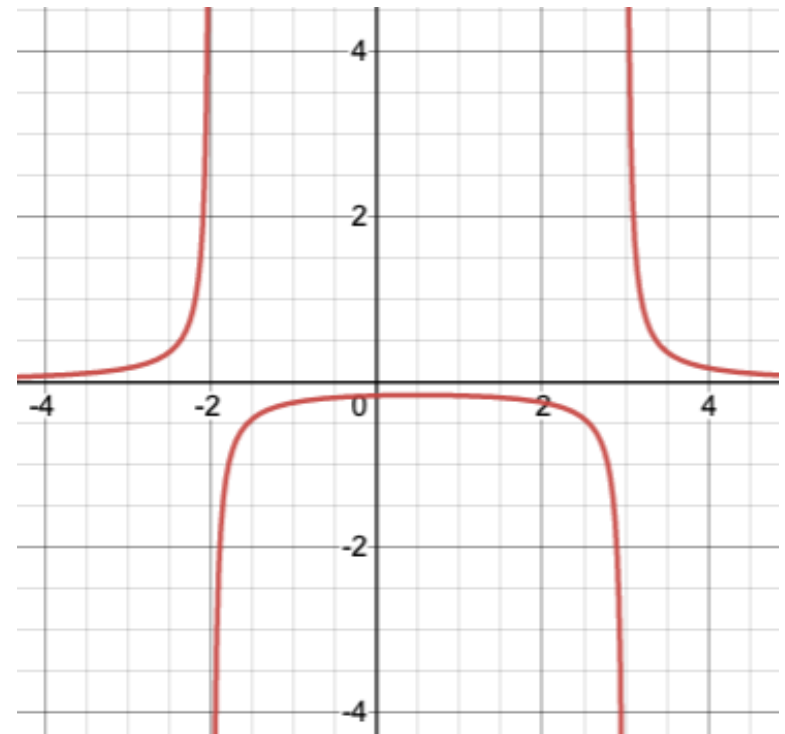
What's the behavior of the graph as it approaches the vertical asymptote from right/left?

$$\lim_{x \rightarrow -2^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -2^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = +\infty$$



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Domain

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$$f(x) = \frac{1}{x+2}$$

$$\lim_{x \rightarrow -2^-} f(x) = -\infty \quad \text{From left}$$

$$\lim_{x \rightarrow -2^+} f(x) = +\infty \quad \text{From right}$$

Practice - Describe the behavior at values of x not in domain.

1. $f(x) = \frac{-1}{x-2}$

2. $f(x) = \frac{1}{x^2-9}$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = +\infty$$

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -3^+} f(x) = -\infty$$

$$\lim_{x \rightarrow -3^-} f(x) = +\infty$$

7 Graphing Advanced Rational Equations

Domain

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$f(x) = \frac{1}{x}$ This is the original function
that we will be transforming.

What is $h(x)$ in terms of $f(x)$?

$$h(x) = \frac{1}{x-2} = f(x-2)$$

$$h(x) = \frac{3}{x+5} = 3 \cdot f(x+5)$$

$$h(x) = \frac{-1}{x-2} + 3 = -f(x-2) + 3$$

$$h(x) = \frac{3x-7}{x-2}$$

$$h(x) = 3 - \frac{1}{x-2} = 3 - f(x-2)$$

Let's look at the behavior near the vertical asymptotes.

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Behavior around asymptotes

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$$h(x) = \frac{1}{x-2} = f(x-2)$$

$$\lim_{x \rightarrow 2^+} h(x) = +\infty$$

$$\lim_{x \rightarrow 2^-} h(x) = -\infty$$

$$h(x) = \frac{3}{x+5} = 3 \cdot f(x+5)$$

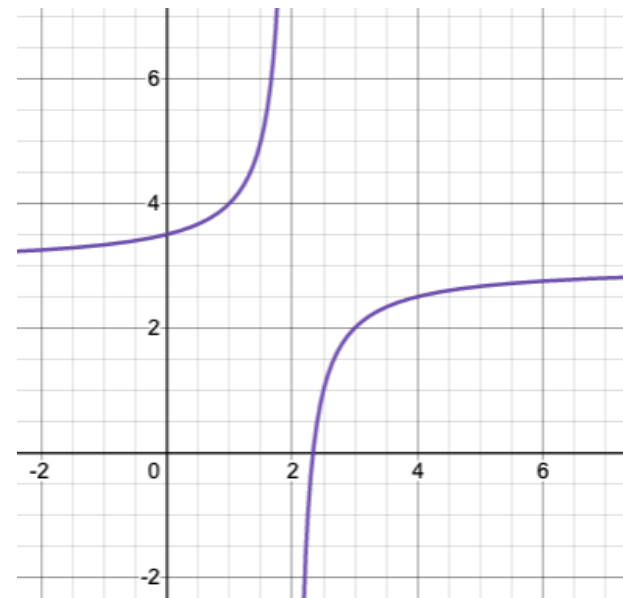
$$\lim_{x \rightarrow -5^+} h(x) = +\infty$$

$$\lim_{x \rightarrow -5^-} h(x) = -\infty$$

$$h(x) = \frac{-1}{x-2} + 3 = -f(x-2) + 3$$

$$\lim_{x \rightarrow 2^+} h(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} h(x) = +\infty$$



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Behavior around asymptotes

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$$h(x) = \frac{3}{x+5} = 3 \cdot f(x+5)$$

$$\lim_{x \rightarrow -5^+} h(x) = +\infty$$

$$\lim_{x \rightarrow -5^-} h(x) = -\infty$$

$$h(x) = \frac{-1}{x-2} + 3 = -f(x-2) + 3$$

$$\lim_{x \rightarrow 2^+} h(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} h(x) = +\infty$$

Practice - Transform into $f(x) = 1/x$ and describe behavior

1. $h(x) = \frac{-3}{x+7}$

$$h(x) = -3f(x+7)$$

$$\lim_{x \rightarrow -7^+} h(x) = -\infty$$

$$\lim_{x \rightarrow -7^-} h(x) = +\infty$$

2. $h(x) = \frac{-4x+3}{x-2}$

$$h(x) = -5f(x-2) - 4$$

$$\lim_{x \rightarrow 2^+} h(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} h(x) = +\infty$$

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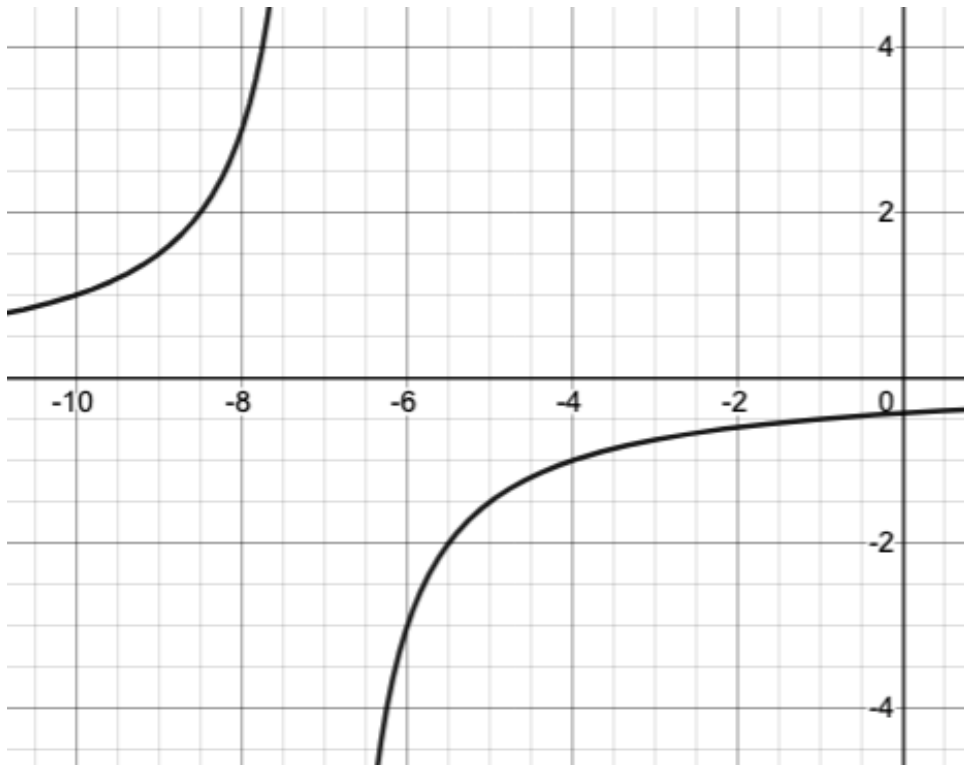
Let's look at it graphically

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$$1. \ h(x) = \frac{-3}{x+7}$$

$$h(x) = -3f(x+7)$$

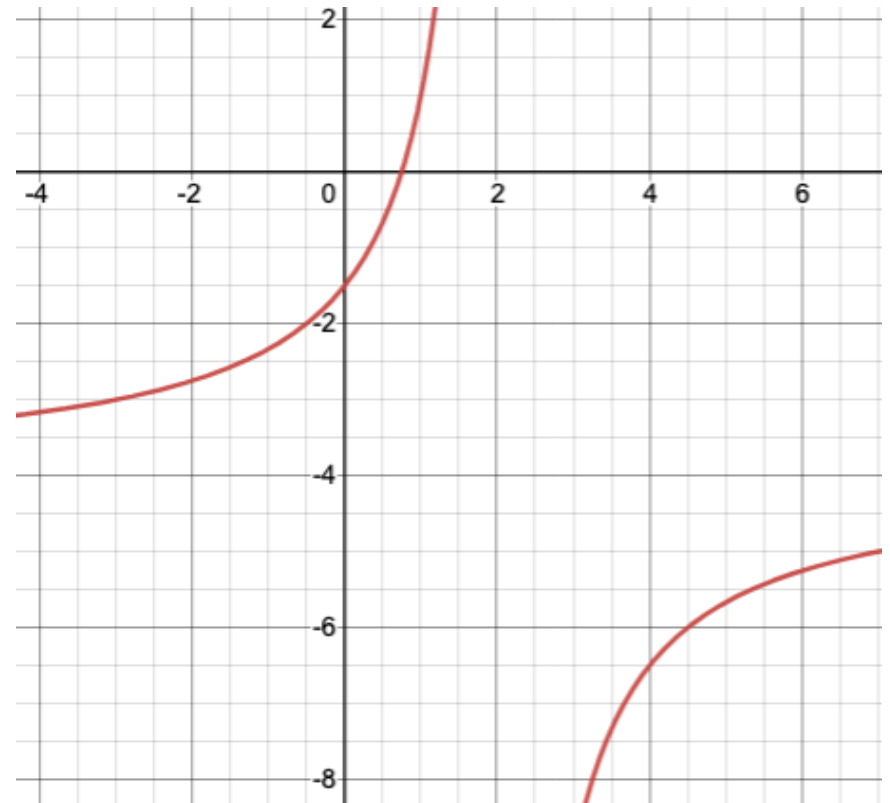
$$\lim_{x \rightarrow -7^+} h(x) = -\infty \quad \lim_{x \rightarrow -7^-} h(x) = +\infty$$



$$2. \ h(x) = \frac{-4x+3}{x-2}$$

$$h(x) = -5f(x-2) - 4$$

$$\lim_{x \rightarrow 2^+} h(x) = -\infty \quad \lim_{x \rightarrow 2^-} h(x) = +\infty$$



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Asymptotes and Intercepts

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Find the asymptotes and intercepts of the function.
Sketch the graph.

$$f(x) = \frac{2x^2 + 5x + 2}{x^2 - 9}$$

Vertical asymptotes

$$x = 3, x = -3$$

Horizontal asymptotes

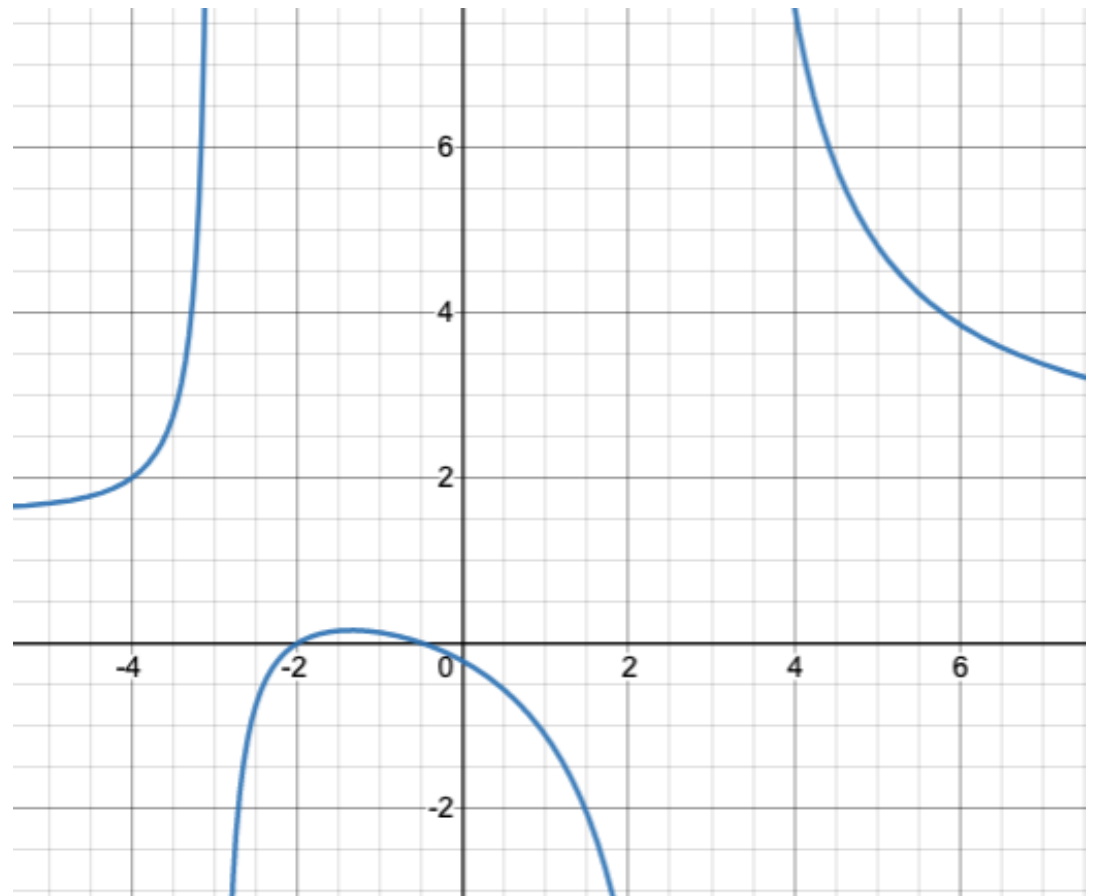
$$y = 2$$

x-intercepts

$$x = -2, x = -\frac{1}{2}$$

y-intercepts

$$y = -\frac{2}{9}$$



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Asymptotes and Intercepts

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What about the horizontal asymptote of:

$$f(x) = \frac{2x^2 + 3x + 4}{3x^3 + 6x^2 - x + 7}$$

$$\lim_{x \rightarrow \infty} f(x) = 0 \quad \text{So horizontal asymptote } y = 0$$

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Asymptotes and Intercepts

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$$f(x) = \frac{2x^3 + 3x^2 - 4x - 6}{x^4 - 16}$$

Vertical asymptotes

$$x = 2, -2$$

Horizontal asymptotes

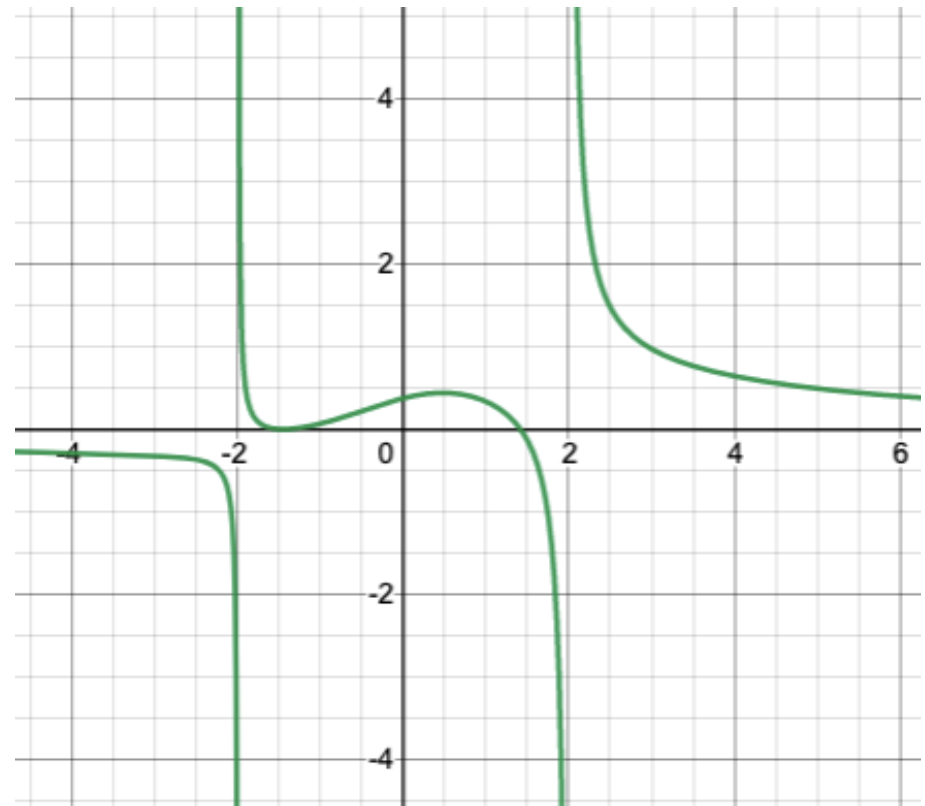
$$y = 0$$

x-intercepts

$$x = -\frac{3}{2}, \pm \sqrt{2}$$

y-intercepts

$$y = \frac{3}{8}$$



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Finding Asymptotes

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Vertical asymptotes occur at the zeros of the denominator provided that the zeros are not also zeros of the numerator of equal or greater multiplicity.

$$f(x) = \frac{2x + 1}{x + 3}$$

Asymptotes: $x = -3$

$$f(x) = \frac{2x + 1}{(2x + 1)(x - 3)}$$

$x = 3$

Hole at $x = -\frac{1}{2}$

$$f(x) = \frac{x + 1}{(x + 1)^2}$$

$x = -1$

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Finding Asymptotes

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Find x-intercepts, y-intercepts, asymptotes, and holes.

1. $f(x) = \frac{2x^2 - 1}{x^2 - 4}$ $x - \text{int} : \pm \frac{\sqrt{2}}{2}$ Asymptotes: $x = \pm 2, y = 2$
 $y - \text{int} : \frac{1}{4}$

2. $f(x) = \frac{2x - 1}{4x^2 - 1}$ $x - \text{int} : \text{none}$ Asymptotes: $x = -\frac{1}{2}, y = 0$
 $y - \text{int} : 1$ Hole at $x = \frac{1}{2}$

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Graphing

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$$f(x) = \frac{-1}{6x^2 + x - 1}$$

Vertical asymptotes

$$x = -\frac{1}{2}, x = \frac{1}{3}$$

x-intercepts

none

y-intercepts

$$y = 1$$

Horizontal asymptotes

$$y = 0$$

Behavior near vertical asymptotes

$$-\frac{1}{2}^+ = \infty, -\frac{1}{2}^- = -\infty, \frac{1}{3}^+ = -\infty, \frac{1}{3}^- = \infty$$

